Prob 1: Particle of mass m which moves freely inside an infinite ∞ potⁿ well of width a, $\left(V(x) = \begin{cases} 0 & , 0<x<a \\ \infty & \text{otherwise} \end{cases} \right)$ has the following wave fucⁿ at $t=0$,

$$\psi(x,0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} \sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

A is real constant.

(1) Find the value of A so that $\psi(x,0)$ is normalise to unity.

(2) If measurment of energies are carried out what are the values that will be found & what are the corresponding probabilities. Also calculate the $\langle E \rangle$.

(3) find the wave fucⁿ at time t, $\psi(x,t)$

(4) Determine the prob. of finding the system at time t in the state,

$$\phi(x,t) = \sqrt{\frac{2}{a}} \sin\left(\frac{5\pi x}{a}\right) e^{-\frac{iE_5 t}{\hbar}}$$

Note - $\psi = \sum_n c_n \phi_n = c_1\phi_1 + c_2\phi_2 + c_3\phi_3 + \cdots$

ϕ_1, ϕ_2, ϕ_3 are orthonormal wave fucⁿs.

$$\psi^* = \sum_m c_m^* \phi_m^*$$

$$\psi^*\psi = \sum_m c_m^* \phi_m^* \sum_n c_n \phi_n$$

$$\int_{\text{all space}} |\psi|^2 dx = 1$$

$$\int \sum_m c_m^* \phi_m^* \sum_n c_n \phi_n \, dx = 1$$

$$\sum_m \sum_n c_m^* c_n \int \phi_m^* \phi_n \, dx = 1$$

If $\int \phi_m^* \phi_n \, dx = \delta_{mn}$

So $\sum_m \sum_n C_m^* C_n \delta_{mn} = 1$

If $m \neq n$ then $\delta_{mn} = 0$

$m = n$, $\sum_n |C_n|^2 = 1$

$\psi = \sum_n C_n \phi_n = C_1\phi_1 + C_2\phi_2 + C_3\phi_3 + \cdots + C_n\phi_n$

Prob. of finding the particle in ground state is $= |C_1|^2$

We have to calculate the cofficients.

$\int \phi_n^* \psi \, dx = \int (C_1\phi_1 + C_2\phi_2 + \cdots + C_n\phi_n + \cdots) dx$

$$\boxed{C_n = \int \phi_n^* \psi \, dx}$$

$$\boxed{P_n = |C_n|^2}$$

If a linear combination is given in wavefn. then prob. may be diffent for each st... y upon these cofficients (C_n).

$\sum |C_n|^2 = 1$

Sol^n

$$\psi(x,0) = \frac{A}{\sqrt{a}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}} \sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}} \sin\left(\frac{5\pi x}{a}\right)$$

$$= \frac{\sqrt{2}}{\sqrt{a}} \frac{A}{\sqrt{2}} \sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{2}{a}} \sqrt{\frac{3}{10}} \sin\left(\frac{3\pi x}{a}\right) + \frac{\sqrt{2}}{\sqrt{a}} \sqrt{\frac{1}{10}} \sin\left(\frac{5\pi x}{a}\right)$$

Now, $\sum |C_n|^2 = 1$ $= \frac{A}{\sqrt{2}} \phi_1(x) + \sqrt{\frac{3}{10}} \phi_3(x) + \sqrt{\frac{1}{10a}} \phi_5(x)$

$$\Rightarrow \frac{A}{\sqrt{2}} \left(\sqrt{\frac{2}{a}} \sin\left(\frac{\pi x}{a}\right)\right) + \sqrt{\frac{3}{10}} \left[\sqrt{\frac{2}{a}} \sin\left(\frac{3\pi x}{a}\right)\right] +$$

$$\sqrt{\frac{1}{10}} \left[\sqrt{\frac{2}{a}} \sin\left(\frac{5\pi x}{a}\right)\right]$$